# The Influence of the N/Z Ratio of Neutron Richness

on the Subbarrier Fusion Dynamics

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## Abstract

The isotopic dependence of the interacting potential and fusion cross sections in 53 isotopic reactions  ${}^{AI}C + {}^{A2}Si$ ,  ${}^{AI}O + {}^{A2}Mg$ ,  ${}^{AI}O + {}^{A2}Si$ ,  ${}^{AI}Si + {}^{A2}Si$ ,  ${}^{AI}Mg + {}^{A2}S$ ,  ${}^{AI}Si + {}^{A2}Ni$ ,  ${}^{AI}Ca + {}^{A2}Ca$ ,  ${}^{AI}S + {}^{A2}Ni$ ,  ${}^{AI}Ar + {}^{A2}Ni$ ,  ${}^{AI}Ca + {}^{A2}Ni$ ,  ${}^{AI}Ni + {}^{A2}Ni$ ,  ${}^{AI}Ti + {}^{A2}Ni$ ,  ${}^{AI}Ca + {}^{A2}Ti$  and  ${}^{AI}O + {}^{A2}Sm$  with  $1 \le N/Z < 1.6$  are studied in systematically. An analytical form obtained in the framework of the Extended Thomas-Fermi method is used to calculate the nuclear part of the interacting potential in single barrier passing model. We investigated the relationships between the changes of potential energy and fusion cross section as a function of (N/Z - 1) quantity.

Keywords: Potential barrier, barrier position and height, fusion cross section

## 1. Introduction

The study of fusion barrier and fusion cross sections has renewed attention in recent decades, due to the fusion of colliding nuclei with neutron-rich/-deficient content at the extreme of isospin plane and also their isotopic behavior [1]. Such studies can be very useful to predict the properties of new and superheavy elements (SHE) that are produced in the fusion process and are not available at present. In general, the formation of neutron rich, SHE and super SHE elements [2] in the universe can be seen far from the  $\beta$ -stability line. Moreover, by increasing the neutron in interacting systems and decreasing the barrier height, fusion cross section enhance, compared to the compound nucleus near the stability line in nuclear chart.

In the first attempt, R. K. Puri *et al.* presented a unified description of isotopic dependence of fusion barriers and cross-sections of neutron content  $(0.5 \le N/Z \le 2.0)$  systems for Ca-Ni colliding series, by employing several different theoretical models such as Skyrme energy density model and proximity potential, as well as parameterized potentials [3]. In order to be systematic study on the isotopic dependence of fusion barriers and cross-sections, O. N. Ghodsi *et al* analyzed fusion dynamics for both proton- and neutron-rich 125 systems, to be in three ranges (i)  $0.5 \le N/Z \le 1$ , (ii)  $1 \le N/Z \le 1.6$ , and (iii)  $0.5 \le N/Z \le 1.6$  [4]. They calculated the nuclear potentials and fusion cross sections by using the proximity formalism and Wong model. Their obtained results showed that although the variations trend of barrier quantities, follow a linear dependence as a function of (N/Z-I) with conditions of  $0.5 \le N/Z \le 1$ , and  $1 \le N/Z \le 1.6$ , these values follow a nonlinear second-order behavior in the whole range  $0.5 \le N/Z \le 1.6$ .

In this study, we shall here attempt to highlight the influence of the N/Z ratio of neutron richness on the subbarrier fusion probabilities of 53 chosen reactions by employing the analytical form obtained in the framework of the Extended Thomas-Fermi method as a form of nuclear potential. The

calculated fusion cross sections are obtained by numerically solving the Schr ödinger equation in one single barrier passing model.

The paper is organized as follows: in section 2, after describing the formulation of the potential model for fusion reactions in brief, we will present about the nuclear part of the total potential. Section 3 yields an analysis of the isotopic dependence of the fusion barriers and cross sections, whereas we will draw the conclusions from the present work in the final section.

#### 2. Formalism

In the present study, the one dimensional barrier passing model is used to study the fusion cross section

$$\sigma_F(E) = \frac{\pi}{k^2} \sum_l (2l+1) P_l(E),$$
(1)

where  $P_l(E)$  is the penetrability for the *l*-wave scattering determined by numerically solving the Schrödinger equation for the radial motion with an appropriately chosen potential [5,6,7]. With the assumption that  $E_{c.m.} >> V_B$ , the preceding formula reduces to the well-known sharp cutoff formula

$$\sigma_F(E) = 10\pi R_B^2 \left( 1 - \frac{V_B}{E_{c.m.}} \right).$$
(2)

By adding the Coulomb potential to a nuclear part, we can compute the total potential for spherical colliding systems as

$$V(r) = V_N(r) + V_C(r).$$
 (3)

In this study the nuclear potentials  $V_N(r)$  are taken to be of the following form [8]

$$V_N(r) = -V_0 + \alpha (r - r_0)^2 \qquad \text{for } r \langle r_0 \\ = -V_0 \exp\left[-\alpha (r - r_0)^2\right] \qquad \text{for } r \geq r_0$$
(4)

In the framework of mean field theory, where, as before, the minimum distance  $r_0$  and the depth of the potential  $V_0$  are direct results of our semi-classical calculation. These two quantities are parameterized in the following way as functions of the masses  $A_P, A_T$  and reduced isospins  $I_i = (A_i - 2Z_i)/A_i$  of target and projectile nuclei

$$r_0(A_P, A_T) = r_0\left(A_P^{\frac{1}{3}} + A_T^{\frac{1}{3}}\right) + b$$
(5)

$$V_0(A_P, A_T, I_P, I_T) = \upsilon_0 \left[ 1 + \kappa (I_P + I_T) \right] \frac{A_P^{\frac{1}{3}} A_T^{\frac{1}{3}}}{A_P^{\frac{1}{3}} + A_T^{\frac{1}{3}}},$$
(6)

where the values of the parameters in Table (1) entering into these expressions can be adjusted to the semi-classical extended Thomas Fermi (ETF) nuclear potential [9].

Table 1. Parameters of the mean field (Mfield) potential.

$$\frac{r_0(fm) \qquad b(fm) \qquad \upsilon_0(MeV) \qquad \kappa \qquad \alpha(fm^{-2})}{1.202 \qquad -2.4 \qquad -49.07 \qquad -0.4734 \qquad 0.173}$$

#### 3. Isotopic Analysis

The quality of our parameterized fusion barrier can be judged by analyzing the percentage deviation of the heights and positions of the fusion barrier with respect to their corresponding values for the N=Z cases as

$$\Delta R_{B}(\%) = \frac{R_{B} - R_{B}^{0}}{R_{B}^{0}} \times 100, \text{ and } \Delta V_{B}(\%) = \frac{V_{B} - V_{B}^{0}}{V_{B}^{0}} \times 100, \qquad (7)$$

where  $R_B^0$  and  $V_B^0$  are the positions and heights of the barrier for the N = Z cases. In all systems, where the values of  $R_B^0$  and  $V_B^0$  are not available, a straight-line interpolation is used between the known points to compute the  $\Delta R_B(\%)$  and  $\Delta V_B(\%)$ . The obtained results for percentage difference of  $R_B$  and  $V_B$  for the mean field (Mfield) potential, are plotted in Fig.1. (a) and (b), in which the increase of barrier positions and decrease of barrier heights are both linear. We have parameterized these processes by following equations,

$$\Delta R_B(\%) = \alpha \left(\frac{N}{Z} - 1\right)$$
, and  $\Delta V_B(\%) = \beta \left(\frac{N}{Z} - 1\right)$ , (8)

where the values of the constants  $\alpha$  and  $\beta$  for Mfield potential have been listed in Table (2). We expect that with increase of neutron in the interaction nuclei of fourteen groups of colliding systems, the nuclear attractive force increases and therefore the heights of barrier decrease.



Fig.1. (a) The percentage increase of  $R_B$  and (b) The percentage decrease of  $V_B$ 

To study the increase of nuclear potential with addition of neutron, like Eqs. (8), we have parameterized the trend of  $V_N$  variations. This process has been parameterized by following relations,

$$\Delta V_N(\%) = \gamma \left(\frac{N}{Z} - 1\right) , \qquad (9)$$

where the values of  $\gamma$  for the Mfield potential have also been listed in Table (2). In Fig.2. (a), we have shown the total interaction potential by using Mfield potential for  ${}^{AI}Ni + {}^{A2}Ni$  system, with

increasing neutron in fusion reactions. Therefore, it is predictable that the increase of attractive force could be dominated the increase of the repulsive force.



Fig.2. (a) Ion-Ion potentials (b) The fusion cross sections for  ${}^{A1}Ni + {}^{A2}Ni$  system

Knowledge of the shape of the potential, as well as the barrier position and the height, allows one to calculate the fusion cross section at a microscopic level. In Fig.2. (b), the round dot, dash and dash dot curves based on the calculated fusion cross sections are for  ${}^{58}Ni+{}^{58}Ni,{}^{58}Ni+{}^{64}Ni$  and  ${}^{64}Ni+{}^{64}Ni$  respectively. Moreover, with addition of neutron in these fusion reactions, the fusion probabilities are increased. The percentage difference for fusion cross section is given by the following relation,

$$\Delta \sigma_F(\%) = \frac{\sigma_F(E_{c.m.}^0) - \sigma_F^0(E_{c.m.}^0)}{\sigma_F^0(E_{c.m.}^0)} \times 100 \quad , \tag{10}$$

where  $E_{c.m.}^0 = E_{c.m.}/V_B^0$ . According to the condition  $E_{c.m.} >> V_B$ , we have calculated this percentage difference for two center-of-mass energies  $E_{c.m.}=1.125 V_B^0$  and  $E_{c.m.}=1.375 V_B^0$ . As seen from this figure, the relationship between changes of the fusion cross sections with increasing neutron (or ration N/Z) in four versions of potential are linear. This relation is given by,

$$\Delta \sigma_F(\%) = c \left(\frac{N}{Z} - I\right) , \qquad (11)$$

where the values of constant c for the mean field (Mfield) potential and energies have been listed in Table (2). In order to reduce the barrier height with increasing neutron, one expects an increase of fusion cross section in each of the chosen interaction systems in Fig.3. (a) and (b).

Table 2. The calculated values for fitting parameters for the mean field potential.

Potential Type	α	β	γ	$c(E_{c.m.}^0 = 1.125V_B^0)$	$c(E_{c.m.}^0 = 1.375V_B^0)$
Mean field	8.8897	-8.142	-16.645	86.47	40.906



Fig.3. (a) The percentage increase of  $\sigma_F$  (mb) for  $E_{c.m.} = 1.125 V_B^0$  and (b) for  $E_{c.m.} = 1.375 V_B^0$ 

### 4. Conclusion

In this contribution, we have confirmed that the relation between changes of the barrier position  $\Delta R_B$  and barrier height  $\Delta V_B$  as well as the trend of nuclear potential  $V_N$  (r) in  $r = R_B$  versus N/Z ratio with increasing of the ratio N/Z in fourteen groups of the fusion reactions are linear for the mean field (Mfield) potential. Moreover, increasing of the attractive force due to neutron excess and consequently the reduction the Coulomb barrier height leads to increase of the fusion cross section. As a result, the changes of the fusion cross sections  $\Delta \sigma_F$  with increasing of the ratio N/Z in fourteen groups of the interaction systems are also linear.

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